2017 Enrolment The 3rd

Japan University Examination

Mathematics（Science）

Examination Date: July 2016

（90 min）

Do not open the examination booklet until the starting signal for the exam is given. Please read the following instructions carefully.
Please fill in the examinee no. and name below.

Instructions

１．The booklet contains 7 pages.

２．The answer sheet is one piece of one sided paper.

３．In the case that you notice there are parts in the booklet where the print is not clear or there are missing pages or misplaced pages, or the answer sheet is soiled, raise your hand to report to the invigilator.

４．There are 4 questions to be answered.

５．Fill the examinee no. and name in the answer sheet.

６．Use black pencil to write answers in the designated section in the answer sheet.

７．Memos and calculations can be written on the examination booklet.

８．When the signal to end the exam is given, check again to see that the examinee no. and name is filled in and submit the answer sheet and the examination booklet according to the invigilator's instructions.

Examinee's No.	Name

$\boxed{1}$ Fill in the following blanks from $\boxed{\text{A}}$ to $\boxed{\text{X}}$ with the correct numbers.

(1) If the quadratic equation: $x^2 - 4x - 3 = 0$ has two roots, and the greater one is α, then

$$\alpha = \boxed{\text{A}} + \sqrt{\boxed{\text{B}}}$$

and if the integer part of α is a, the decimal part is b, then

$$a = \boxed{\text{C}},\ b - \frac{3}{b} = \boxed{\text{DE}}$$

(2) If $x = \frac{1}{\sqrt{3}+1}$, $y = \frac{1}{\sqrt{3}-1}$, then

$$x + y = \sqrt{\boxed{\text{F}}},$$

$$xy = \frac{\boxed{\text{G}}}{\boxed{\text{H}}},$$

$$x^3 + y^3 = \frac{\boxed{\text{I}}\sqrt{\boxed{\text{J}}}}{\boxed{\text{K}}}$$

(3) Consider a triangle ABC with $\text{AB} = 5$, $\text{BC} = 2\sqrt{6}$, $\text{CA} = 3$, then

$$\cos\angle\text{BAC} = \frac{\boxed{\text{L}}}{\boxed{\text{M}}}$$

And let S be the area of ABC, and R be the radius of the circumcircle of ABC, then

$$S = \boxed{\text{N}}\sqrt{\boxed{\text{O}}},$$

$$R = \frac{\boxed{\text{P}}\sqrt{\boxed{\text{Q}}}}{\boxed{\text{R}}}$$

(4) Both x and y are real numbers, and

$$2^x = 3,\ \ 4^y = 36$$

$$x = \log_2 \boxed{\text{ S }},\ \ y = \log_2 \boxed{\text{ T }} + \boxed{\text{ U }}$$

In this situation, if $\log_{10}2 = a$, $\log_{10}3 = b$, then

$$x + y = \frac{\boxed{\text{ V }}\, b}{a} + y\,\boxed{\text{ W }}$$

[2] Fill in the following blanks from [A] to [YZ] with the correct numbers.

(1) k is a real constant. Here is the equation for x

$$x^2 + 2(3 - 2k)x + k = 0 \qquad \ldots(*)$$

$(*)$ has equal root, then

$$k = \boxed{\text{A}},\ \frac{\boxed{\text{B}}}{\boxed{\text{C}}}$$

In this situation, if the equal root of $(*)$ is a negative number,

$$x = \boxed{\text{DE}}$$

(2) In the geometric series $\{a_n\}, a_5 = 48,\ a_9 = 768$, then

the first term $a_1 = \boxed{\text{F}}$, and the common ratio is $\boxed{\text{G}}$

and the n[th] term a_n can be written as

$$a_n = \boxed{\text{H}} \cdot \boxed{\text{I}}^{\,n-\boxed{\text{J}}}$$

In this situation,

$$\frac{1}{a_1} + \frac{1}{a_2} + \frac{1}{a_3} + \cdot\cdot\cdot + \frac{1}{a_{10}} = \frac{\boxed{\text{KLM}}}{\boxed{\text{NOP}}}$$

(3) a is a real constant. When $0° \leqq \theta < 360°$, and one of the solutions of the equation for θ:

$$2\sin(\theta + 30°) = a \qquad \ldots(*)$$

is $\theta = 90°$, then

$$a = \sqrt{\boxed{\text{Q}}}$$

In this situation, the other solution of $(*)$(except for $\theta = 90°$) is

$$\theta = \boxed{\text{RS}}°$$

(4) m is a real constant.

Circle C : $x^2 + y^2 - 2x - 8y + 13 = 0$

Straight line l : $mx - y + m - 3 = 0$

The coordinate of the centre of C is $(\boxed{\text{T}}, \boxed{\text{U}})$, and the radius is $\boxed{\text{V}}$.

In addition, C and l intersect at 2 different points, then

$$m > \frac{\boxed{\text{WX}}}{\boxed{\text{YZ}}}$$

3 Fill in the following blanks from ABC to QR with the correct numbers.

There are 18 cards which are colored red, blue and yellow. Each color contains 6 cards which are numbered 1~6 .Once put the 18 cards into a bag and then take out 3 cards at the same time.

(1) There are ABC different kinds of combinations of the 3 cards in total. If the 3 cards are all red, the kinds of combinations should be DE. Then if at least one of the 3 cards is numbered with 1, the kinds of combinations should be FGH.

(2) Suppose A and B represent the 2 cases below:

A : the 3 cards taken out are in the same color

B : the numbers on the 3 cards taken out are consecutive numbers

In addition, a, b fulfill the following conditions:

If A occurs, $a = 1$, and if A doesn't occur, $a = 0$

If B occurs, $b = 1$, and if B doesn't occur, $b = 0$

then,

The probability of $a = 1$ is $\frac{\boxed{\text{I}}}{\boxed{\text{JK}}}$, $b = 1$ is $\frac{\boxed{\text{L}}}{\boxed{\text{MN}}}$.

The probability of $a = 0$, while $b = 0$ as well is $\frac{\boxed{\text{OP}}}{\boxed{\text{QR}}}$.

4 Fill in the following blanks from AB to A' with the correct numbers.

[1] a is a constant. Here are 2 inequalities for x

$x^2 - x - 2 > 0$ …①

$x^2 - (a+4)x+4a \leqq 0$ …②

(1) The solution of inequality ① is

$x <$ AB , C $< x$

(2) If there is only one real number of x that satisfy inequality ②, then

$a =$ D

In this situation, the solution of inequality ② is

$x =$ E

(3) There are 3 integers of x that satisfy both inequality ① and ②, so the range of a should be

FG $< a \leqq$ HI , J $\leqq a <$ K

[2] Both a and b are constants. The parabola of quadratic function: $y = -2x^2 + ax + b$ is named G_1 , and it goes through point (1, −3).

(1) $b = -a - \boxed{\text{L}}$

The coordinate of $G_1's$ apex is

$$\left(\frac{a}{\boxed{\text{M}}},\ \frac{a^2}{\boxed{\text{N}}} - a - \boxed{\text{O}}\right)$$

If G_1 and x-axis intersect at 2 different points, then the range of a shuold be

$$a < \boxed{\text{P}} - \boxed{\text{Q}}\sqrt{\boxed{\text{R}}},\ \boxed{\text{P}} + \boxed{\text{Q}}\sqrt{\boxed{\text{R}}} < a$$

(2) The parabola of quadratic function: $y = 2x^2 - ax - b$ is named G_2. G_1 intersects with y-axis at M, and G_2 intersects with y-axis at N.

If the y-coordinate of M is greater than the y-coordinate of N, the range of a should be

$$a < \boxed{\text{ST}}$$

In this situation, G_1 and x-axis intersect at point A and B

$$\text{AB} = \frac{\boxed{\text{U}}}{\boxed{\text{V}}}\sqrt{a^2 - \boxed{\text{W}}\,a - \boxed{\text{X}}}$$

If AB : MN = 5 : 4, then

$$a = \frac{\boxed{\text{YZ}}}{\boxed{\text{A'}}}$$